International Journal of Scientific & Engineering Research, Volume 9, Issue 5, May-2018
ISSN 2229-5518

Determination of Effectiveness of Inclined Stiffeners of Thin Cylindrical Shell under Uniform Bending

[1] Anyaogu, L, [2] Ezeh, J.C., [3]Ibearugbulem, O.M., [4] Ettu, L.O, [5] Okoroafor, S. U.
1,2,3,4,5Department of Civil Engineering, Federal University of Technology, Owerri, Imo State, Nigeria

Abstract— This research aimed at determination of effectiveness of inclined stiffeners of thin cylindrical shell under uniform bending. The method of solution was carried out by the use of nonlinear large deflection theory and the effect of initial imperfections in the strain-displacement equations was considered. The Ritz method was used to determine the buckling stress parameter of the shell. Numerical examples were carried by varying the angle of inclination of the stiffeners at different imperfect ratios with other properties like: flexural rigidity and torsional rigidity of the stiffeners, deflection parameters, internal pressure and radius of curvature of the shell being kept constant. The results showed that 10^0 inclined stiffeners are the most effective with its maximum critical buckling stress at imperfect ratio of 0.5. While 45^0 inclined stiffener is the least effective with its least critical buckling stress at imperfect ratio of 0.1. With reference to the results obtained in this research, engineers designing cylindrical shell structures with the aim of providing resistance to buckling would be able to select suitable inclined stiffeners for the structure under uniform bending.

Index Terms— Thin cylindrical shell, buckling, stress, uniform bending, the Ritz, imperfect ratio, inclined stiffeners, effectiveness.

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1 Introduction

The The design of cylindrical shell structures depends on a large number of factors, namely the economic aspects, material availability, response of each structure of the system to static and dynamic loads, temperature effects and so on. The designer is interested in arriving at an optimum design taking into considerations all these factors [1]. The analysis of the structure is normally concerned with the determination of behaviour of the structure or the elements of the structure under the action of external loads. It explains the response of the structure when subjected to external loads and /or temperature changes. In other words, if the external loads are known, the deformation pattern and internal stress distribution in the structure can be determined. Also, the nature of equilibrium of the structure (stable or unstable equilibrium) shall be determined. The understanding of those responses of the structure is necessary for design of safe structure [1].

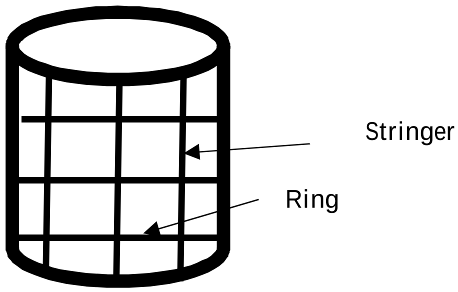
Cylindrical shell structures can fail either by yielding of buckling. The collapse of the structures precipitated by buckling is often a more serious problem than fracture or yielding. Buckling sometime occurs suddenly without warning causing a catastrophic failure.

Fracture or yielding, on the other hand, can also produce failure, but the elasticity of the material permits a redistribution of the stresses often allowing a progressive collapse rather than a sudden complete collapse characteristic of buckling. Once buckling is initiated within the structure, there is little or no chance of recovery unless the load is suddenly reduced [2]. In fact, buckling phenomenon in cylindrical shell occurs when most of the strain energy which is stored as membrane energy has been converted to bending energy requiring large deformation resulting to catastrophic failure [2]. Hence, the design of thin cylindrical shells should be based on buckling criteria [3]. Buckling behaviour of cylindrical shells (in particular, the critical buckling load) is not accurately predicted by linear elastic equations due to initial imperfections of the shell structure under the action of external loads like uniform bending, uniform axial compression etc. The buckling effect on the cylindrical shell structures can be resisted with incorporation of stiffeners in the shell [4]. The circumferential stiffeners are known as ring while longitudinal stiffeners are called stringers [5]; [6]. Cylindrical shell with stiffeners is shown in Fig.1

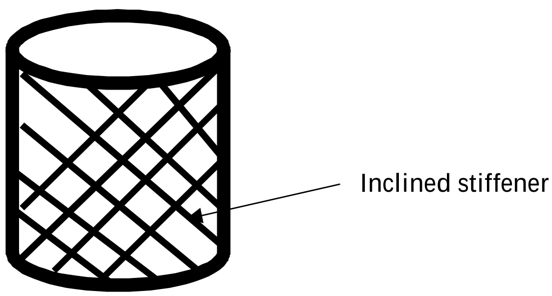
In this work, the Ritz method which was incorporated with imperfections in the shell structures was employed in determining the effectiveness of inclined stiffeners of internally pressurized thin cylindrical shell under uniform bending. This was achieved by assuming the displacement function of the shell. Its stress function was obtained from the assumed displacement function from the compatibility equation which was carried out by non linear large deflection theory. The expression of the stored energy in the shell as well as work done by the external load was obtained using both the stress and displacement functions. The large deflection terms, effect of imperfection in the strain displacement and the external load were considered in the formulation of total strain energy of the imperfect shell. The resulted total strain energy was minimized using the Ritz method to determine the equation for obtaining the buckling stress values of the shell.

· Ezeh, J.C. is A Professor in Civil Engineering Department in Federal University of Technology Owerri Imo State, Nigeria. E-mail: engrlewy@gmail.com.
· Co-Authors are lecturers in Civil Engineering Department in Federal University of Technology Owerri Imo State, Nigeria. E-mail: solomonokoroafor@ymail.com

ISSN 2229-5518



(a)



(b)

Fig 1: Cylindrical shell (a) stiffened with rings and stringers (b) stiffened with inclined stiffeners

2.0 Derivation of Buckling Stress Parameter of Thin Cylindrical Shell under Uniform Bending

The buckling stress parameter was derived thus; the deflection function was assumed first, then the stress function was obtained from the compatibility equation which was carried out by the non-linear large deflection theory. The expression for the stored energy in the shell and stiffeners as well as work done by the external loads (i.e. uniform bending) was obtained using the assumed deflection and stress functions. The large deflection terms, the effect of the imperfection in the strain displacement and strain energy equations of the shell, shell stiffeners and external loads were considered in the formulation of total strain energy for each type of the cylindrical shells. The resulted total strain energy for each type of cylindrical shells was minimized using the Ritz method.
The equation obtained after minimization using the Ritz method is the governing equation for computing the buckling stress value of the cylindrical shell.

2.1 2.1 Energy Expression for the cylindrical shell

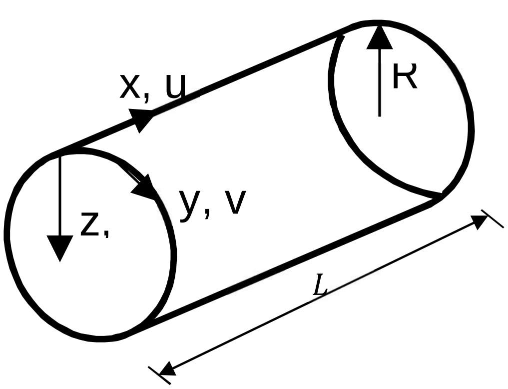


Fig. 2: Coordinates and Displacement Components of a point on the Middle- surface of the shell.
Let x and y be the axial and circumferential axis in the median surface of the undeformed cylindrical shell as shown in Fig. 1, w is the total radial deflection and w0 represents the initial radial deflection. From the theory of elasticity, the strain – displacement relations of the cylindrical shell are as expressed in Eqns. (1a), (1b) and (1c) respectively.

$$\epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{dw_0}{\partial x}\right)^2 \qquad (1a)$$

$$\epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \frac{w - w_0}{R} \qquad (1b)$$

$$\epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}.\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}.\frac{\partial w_0}{\partial y} \qquad (1c)$$

The stresses and strains in the middle surface of the shell in the case of plane stress are related to each other by the following equations.

$$\sigma_x = \frac{E}{1-\mu^2}\left(\epsilon_x + \mu\,\epsilon_y\right) \qquad (2a)$$

$$\sigma_y = \frac{E}{1-\mu^2}\left(\epsilon_y + \mu\,\epsilon_x\right) \qquad (2b)$$

$$\sigma_{xy} = \frac{E}{2(1+\mu)}\epsilon_{xy} \qquad (2c)$$

Substituting Eqns. (1a), (1b) and (1c) into their related equations in Eqns. (2a), (2b) and (2c), the followings were obtained;

$$\sigma_x = \frac{E}{1-\mu^2}\left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \mu\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \left(\frac{w-w_0}{R}\right)\right]\right\} \qquad (3a)$$

$$\sigma_y = \frac{E}{1-\mu^2}\left\{\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \left(\frac{w-w_0}{R}\right) + \mu\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]\right\} \qquad (3b)$$

$$\sigma_{xy} = \frac{E}{2(1-\mu^2)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}.\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}.\frac{\partial w_0}{\partial y}\right] \qquad (3c)$$

For plane stress state, the non-zero components of stress tensor, $\sigma_x, \sigma_y, \sigma_{xy}$ satisfied the following equilibrium using Airy stress function F.

$$\sigma_x = \frac{\partial^2 F}{\partial y^2};\ \sigma_y = \frac{\partial^2 F}{\partial x^2};\ \sigma_{xy} = \frac{-\partial^2 F}{\partial x \partial y} \qquad (4)$$

Eliminating variables u and v in Eqns. (3) and (4), the relation between stress function F and radial component displacement, w was expressed as follows:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 F = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}.\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial x^2}.\frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \frac{1}{R}\frac{\partial^2 w_0}{\partial x^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\right] \quad (5a)$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is called Laplace operator.

$$(\nabla^2)^2 F = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}.\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial x^2}.\frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \frac{1}{R}\frac{\partial^2 w_0}{\partial x^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\right] \quad (5b)$$

For simplicity, w was assumed to be proportional to w_0. Thus,

$$Л = \frac{w_0}{w} \qquad (6)$$

Where Л is called imperfection ratio and it is independent of x and y.
With the expression from Eqns (5b) and (6), the compatibility equation was expressed as;

$$\left(\frac{1}{1-Л}\right)\nabla^4 F = E(1+Л)\left[\left(\frac{\partial^2 w}{\partial x \partial y^2}\right)^2 - \frac{\partial^2 w}{\partial x^2}.\frac{\partial^2 w}{\partial y^2}\right] - \frac{E}{R}\frac{\partial^2 w}{\partial x^2} \qquad (7)$$

Where ∇^4 is called Bilharmonic operator.

Equation (7) is the compatibility equation of perfect thin cylindrical shell.
The strain energy of isotropic medium referred to arbitrary orthogonal coordinates was expressed as:

$$U = \frac{1}{2}\iiint_{vol}\sigma_{ij}\epsilon_{ij}dvol = \frac{1}{2}\iiint_{vol}\left[\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_{xy}2\epsilon_{xy} + \sigma_{xz}2\epsilon_{xz} + \sigma_{yz}2\epsilon_{yz}\right]dxdydz \qquad (8a)$$

Substituting Eqns. 1(a-c), 2(a-c), 3(a-c) and 4 into Eqn. (8a), we have expressions stated in Eqns. (8) and (9) respectively:

i. The extensional strain energy in the shell. This was expressed as;

$$U_e = \frac{h}{2E}\int_0^L\int_0^{2\pi R}\left\{\left(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2}\right)^2 + 2(1+\mu)\left[\left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 - \frac{\partial^2 F}{\partial x^2}.\frac{\partial^2 F}{\partial y^2}\right]\right\}dxdy \qquad (8)$$

ii The potential due to the internal pressure, p of the cylindrical shell

$$U_p = \int_0^L\int_0^{2\pi R} p(w - w_0)\,dxdy \qquad (9)$$

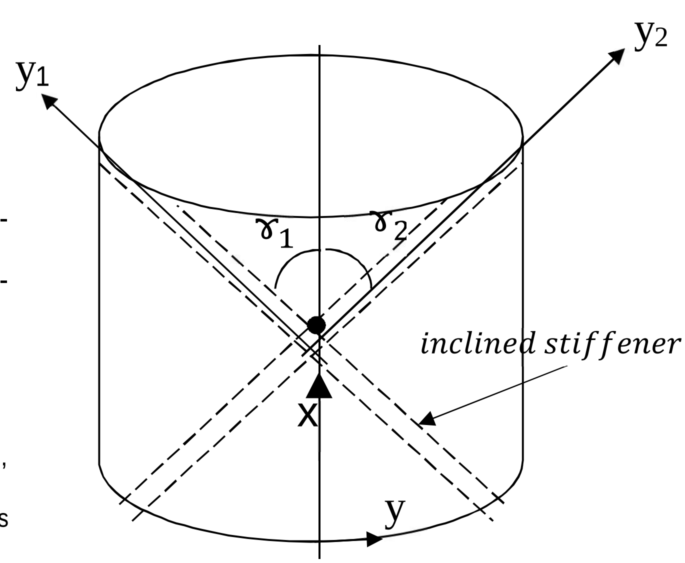
iii. The potential due to edge bending of the shell

As a result of the eccentric loading of the shell, the potential due to the edge bending of the shell is the product of applied bending force and the length in the direction of bending. This expressed as:

$$U_m = \frac{-\sigma_b h}{E}\int_0^L\int_0^{2\pi R}\left[cos\frac{y}{R}\left\{\left(\frac{\partial^2 F}{\partial y^2} - \mu\frac{\partial^2 F}{\partial x^2}\right) - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right\}\right]dxdy \quad (10)$$

Where σ_b = applied peak bending stress

The bending strain energy of stiffeners. Considering Fig. 3, the stiffeners were assumed parallel with the y_1, $y_2$coordinates lines and the principal direction of the cylindrical shell coincide with x, y, lines.



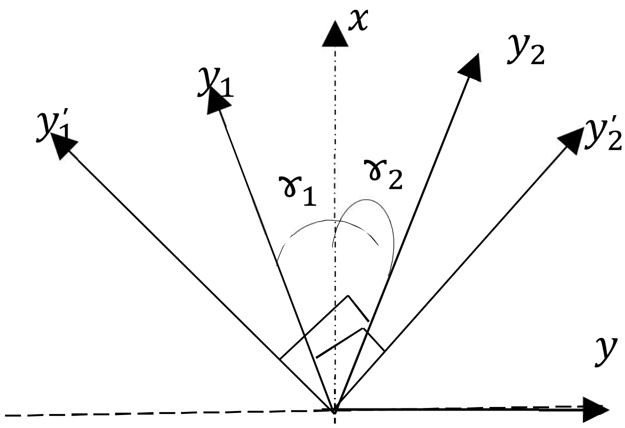


Fig. 3: The coordinate system of the stiffeners of the cylindrical shells and stiffeners

The subscript k was used for k^{th}stiffnener, which is inclined at an angle, $ɤ_1$ with generator of the cylinders and is parallel with y_1-line and normal to y_2''– line. Hence, the bending strain energy in the k^{th} stiffener is

$$U_{b,k} = \sum_{k=1}^{N_k} \frac{E_k I_k}{2} \int_0^{L_k} \left(\frac{\partial^2 w}{\partial y_1^2} - \frac{\partial^2 w_0}{\partial y_1^2} \right)^2_{y_2'=0} dy_1 \qquad (11)$$

Where N_k denotes the number of the stiffeners in $ɤ_1$ – direction. $E_k I_k$ represents the flexural rigidity of the k^{th}stiffener. The limit L_k is the length of the stiffener in $ɤ_1$– direction.

Similarly, the bending strain energy in the j^{th} stiffener which is parallel with y_2– line and nomal to y_1'- line as shown in Fig. 2.

$$U_{b,j} = \sum_{j=1}^{N_j} \frac{E_j I_j}{2} \int_0^{L_j} \left(\frac{\partial^2 w}{\partial y_2^2} - \frac{\partial^2 w_0}{\partial y_2^2} \right)^2_{y_1'=0} dy_2 \qquad (12)$$

The subscript j was used for j^{th} stiffener which is inclined at angle of $ɤ_2$ with the generator of the cylinder. Where N_j is the number of the stiffeners in $ɤ_2$– direction.

$E_j I_j$ represents the flexural rigidity of the j^{th} stiffeners. The limit L_j is length of the stiffener in $ɤ_2$– direction.

$$U_{T,k} = \sum_{k=1}^{N_k} \frac{G_k J_k}{2} \int_0^{L_k} \left[\frac{\partial^2 (w - w_0)}{\partial y_1 \partial y_2'} \right]^2_{y_2'=0} dy_1 \qquad (13)$$

$$U_{T,j} = \sum_{j=1}^{N_j} \frac{G_j J_j}{2} \int_0^{L_j} \left[\frac{\partial^2 (w - w_0)}{\partial y_1 \partial y_2'} \right]^2_{y_1'=0} dy_2 \qquad (14)$$

Where G J represents the torsional rigidity of stiffeners, with subscript j representing stiffeners in $ɤ_2$ - direction and subscript k is for stiffeners in $ɤ_1$- direction. In this analysis, the inclined angles, $ɤ_1$ and $ɤ_2$ are considered in axial symmetry for inclined stiffeners.

The deflection shape of the cylindrical shell under uniform bending was assumed as:

$$w = f_1 + cos^2({}^{y}/_{2R}) \left[f_2 \cos \frac{mx}{R} cos \frac{ny}{R} + f_3 cos \frac{2mx}{R} + f_4 \cos \frac{2ny}{R} \right] \qquad (15)$$

Where m and n are the numbers of waves in axial and circumferential directions respectively. Using compatibility equation in Eqn (7), the corresponding stress function for cylindrical shell under uniform bending:

$$F = -\frac{\sigma}{2} y^2 + \sigma_b R^2 \cos \frac{y}{R} + \frac{1}{2} \frac{PR}{h} x^2 + a_{11} \cos \frac{mx}{R} \cos \frac{ny}{R} + a_{22} \cos \frac{2mx}{R} \cos \frac{2ny}{R} + a_{20} \cos \frac{2mx}{R} + a_{02} \cos \frac{2ny}{R} + a_{31} \cos \frac{3mx}{R} \cos \frac{ny}{R} + a_{13} \cos \frac{mx}{R} \cos \frac{3ny}{R} \qquad (16)$$

Where σ and σ_b are the average axial and peak bending stresses, respectively and are positive for compression.

Substituting Eqn (16) into Eqn (7) and minimizing the resulting equation, the coefficients $a_{11}, a_{22}, a_{02}, a_{20}, a_{31}, a_{13}$ in Eqn. (16) were determined in terms of f_2, f_3, and f_4 as shown in Eqns.17(a-f)

$$\forall_{20}^* = \frac{a_{20}}{Eh^2} = \frac{1}{256} [32 Ƅ_4^* \beta (1 - Л) - \left(3\bar{µ}^2 + \frac{1}{m^2} \right) Ƅ_2^{*2} (1 - Л^2) \qquad (17a)$$

$$\forall_{02}^* = \frac{a_{02}}{Eh^2} = -(1 - Л^2) \frac{3Ƅ_2^{*2}}{256\bar{µ}^2} \qquad (17b)$$

$$\forall_{11}^* = \frac{a_{11}}{Eh^2} = -\frac{(1 - Л^2) \left[\left(12\bar{µ}^2 + \frac{1}{m^2} \right) (Ƅ_3^* + Ƅ_4^*) Ƅ_2^* \right] - 8(1 - Л)\beta Ƅ_2^*}{16(1 + \bar{µ}^2)^2} \qquad (17c)$$

$$\forall_{22}^* = \frac{a_{22}}{Eh^2} = -(1 - Л^2) \left[\frac{\frac{Ƅ^{*2}}{m^2} + Ƅ_3^* Ƅ_4^* \left(96\bar{µ}^2 + {}^{8}/_{m^2} \right)}{256(1 + \bar{µ}^2)^2} \right] \qquad (17d)$$

$$\forall_{31}^* = \frac{a_{31}}{Eh^2} = -\frac{\left(12\bar{µ}^2 + \frac{9}{m^2} \right)}{16(9 + \bar{µ}^2)^2} Ƅ_2^* Ƅ_4^* (1 - Л^2) \qquad (17e)$$

$$\forall_{13}^* = \frac{\left(12\bar{µ}^2 + \frac{1}{m^2} \right)}{16(9\bar{µ}^2 + 1)^2} Ƅ_2^* Ƅ_4^* (1 - Л^2) \qquad (17f)$$

Where

$$\bar{µ} = \frac{n}{m}, \beta = \frac{R}{m^2 h}, \quad Ƅ_1^* = \frac{f_i}{h}, i = 2,3,4 \quad \bar{\sigma} = \frac{\sigma R}{Eh}$$

$\bar{\mu}$ is called wavelength ratio in axial and circumferential direction

3.0 Expression of Total Potential for Cylindrical Shell with inclined stiffeners Subjected to Internal Pressure and uniform bending

The total potential of the system, $\prod$ is the sum of the strain energy and the potential of the applied loads. Thus,

$$\prod = U_e + U_b + U_m + U_P + U_{b,k} + U_{b,j} + U_{T,k} + U_{T,j} \quad (18)$$

ISSN 2229-5518

3.1 Minimization of Total Potential Energy, $\overline{\prod}$, of Internally Pressurized Thin Cylindrical Shell Subjected to Bending

The non-dimensional form of the total potential of the system is express as shown in Eqn (18b)

$$\overline{\prod} = \bar{U}_e + \bar{U}_b + \bar{U}_m + \bar{U}_P + \bar{U}_{b,k} + \bar{U}_{b,j} + \bar{U}_{T,k} + \bar{U}_{T,j} \quad (18b)$$

The total potential energy of the internally pressurized cylindrical shell subjected bending must be a minimum when the structure is in equilibrium. The minimization of the non-dimensional form of the total energy, $\overline{\prod}$ is as expressed in Eqn (19)

$$\frac{\partial\overline{\prod}}{\partial \text{ɓ}_2^*} = 0, \quad \frac{\partial\overline{\prod}}{\partial \text{ɓ}_3^*} = 0, \quad \frac{\partial\overline{\prod}}{\partial \text{ɓ}_4^*} = 0 \qquad (19)$$

Evaluation of $\frac{\partial \bar{U}_2}{\partial \text{ɓ}_2^*} = 0$, $\frac{\partial \bar{U}_2}{\partial \text{ɓ}_3^*} = 0$, $\frac{\partial \bar{U}_2}{\partial \text{ɓ}_4^*} = 0$ yielded Eqns (20) - (22):

$$\frac{\bar{\vartheta}_1\beta}{1-\text{Л}} = \bar{A}_1 + \beta^2(\bar{A}_2 + \bar{A}_3\lambda + \bar{A}_4\lambda^2) + \text{ɓ}_2^{*2}\bar{A}_5 \qquad (20)$$

$$\frac{\bar{\vartheta}_2\beta}{1-\text{Л}} = \text{฿}_1 + \beta^2\text{฿}_2\lambda^2 + \text{ɓ}_2^{*2}\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda}\right) \qquad (21)$$

$$\bar{\vartheta}_3\frac{\beta}{1-\text{Л}} = \bar{a}_1 + (\bar{a}_2 + \bar{a}_3\lambda^2)\beta^2 + \text{ɓ}_2^{*2}\left(\bar{a}_4 + \frac{\bar{a}_5}{\lambda}\right) \qquad (22)$$

The notations used in Eqns. (20), (21) *and* (22) were defined as follows;

$$\bar{\vartheta}_1 = \frac{3}{2}\bar{\sigma} + \bar{\sigma}_b - \frac{3\bar{\mu}^2\bar{P}}{2} = \frac{3}{2}\bar{\sigma} + \bar{\sigma}_b + \bar{\vartheta}_2 \qquad (23)$$

$$\bar{\vartheta}_2 = -\frac{3\bar{\mu}^2\bar{P}}{2} \qquad (24)$$

$$\bar{\vartheta}_3 = \sigma_b + \frac{3}{2}\bar{\sigma} \qquad (25)$$

$$\bar{A}_1 = \frac{1}{8\bar{A}_2(1-\mu^2)} + \overleftrightarrow{\psi}_1 \qquad (26)$$

$$\bar{A}_2 = \frac{1}{(1+\bar{\mu}^2)^2} \qquad (27)$$

$$\bar{A}_3 - \frac{3}{2}\left[(2+\text{Л})(1+\lambda_1)\bar{\mu}^2\bar{A}_2 + \frac{1}{4}\bar{\mu}^2\lambda_1\right] \qquad (28)$$

$$\bar{A}_4 = (1+\text{Л})\left\{\frac{9(1+\lambda_1)^2\bar{\mu}^4\bar{A}_2}{4} + \frac{9\bar{\mu}^4}{4}\left[\frac{\lambda_1^2}{(9+\bar{\mu}^2)^2} + \frac{1}{(1+9\bar{\mu}^2)^2}\right]\right\} \qquad (29)$$

$$\bar{A}_5 = (1+\text{Л}).\frac{9(1+\bar{\mu}^4)}{256} \qquad (30)$$

$$\overleftrightarrow{\psi}_1 = \sum_{k=1}^{Nk}\frac{\bar{E}_k\bar{I}_k\bar{L}_k}{2}\left[\frac{3}{2}(C_1^4 + 6\bar{\mu}^2C_1^2S_1^2 + \bar{\mu}^4S_1^4)\right] + \sum_{j=1}^{N_j}\frac{\bar{E}_j\bar{I}_j\bar{L}_j}{2}\left[\frac{3}{2}(C_2^4 + 6\bar{\mu}^2C_2^2S_2^2 + \bar{\mu}^4S_2^4)\right] + \sum_{k=1}^{Nk}\frac{3\bar{G}_k\bar{J}_k\bar{L}_k}{8}[(C_1+\bar{\mu}S_1)^2(S_1-\bar{\mu}C_1)^2 + (C_1-\bar{\mu}S_1)^2(S_1+\bar{\mu}C_1)^2] + \sum_{j=1}^{N_j}\frac{3\bar{G}_jJ_j\bar{L}_j}{8}[(C_2+\bar{\mu}S_2)^2(S_2-\bar{\mu}C_2)^2 + (C_2-\bar{\mu}S_2)^2(S_2+\bar{\mu}C_2)^2] \qquad (31)$$

$$\text{฿}_1 = \frac{\bar{\mu}^4}{2(1-\mu^2)} + \overleftrightarrow{\psi}_2 \qquad (32)$$

$$\text{฿}_2 = (1+\text{Л}).\frac{9\bar{\mu}^4}{8(1+\bar{\mu}^2)^2}\lambda_1^2 = (1+\text{Л}).\frac{9\bar{\mu}^4\bar{A}_2\lambda_1^2}{8} \qquad (33)$$

$$\text{฿}_3 = \frac{\text{฿}_2}{4\lambda_1^2}.\left[(\lambda_1+1) + \frac{1}{(1+9\bar{\mu}^2)^2\bar{A}_2}\right] \qquad (34)$$

$$\text{฿}_4 = \frac{-3\bar{\mu}^2}{16}\bar{A}_2 \qquad (35)$$

$$\overleftrightarrow{\psi}_2 = \sum_{k=1}^{Nk}3\bar{E}_k\bar{I}_k\bar{L}_k\bar{\mu}^4S_1^4 + \sum_{j=1}^{N_j}3\bar{E}_j\bar{I}_j\bar{L}_j\bar{\mu}^4S_2^4 + \sum_{k=1}^{Nk}3\bar{G}_k\bar{J}_k\bar{L}_k\bar{\mu}^4C_1^2S_1^2 + \sum_{j=1}^{N_j}3\bar{G}_jJ_j\bar{L}_j\,\bar{\mu}^4C_2^2S_2^2 \qquad (36)$$

$$\bar{a}_1 = \frac{1}{2(1-\mu^2)} + \overleftrightarrow{\psi}_3 \qquad (37)$$

$$\bar{a}_2 = \frac{1}{4} \qquad (38)$$

$$\bar{a}_3 = (1+\text{Л})\frac{36\bar{\mu}^4\bar{A}_2}{32} \qquad (39)$$

$$\bar{a}_4 = \frac{9\bar{\mu}^4}{32}(1+\text{Л})\left[\left(\frac{\lambda_1+1}{\lambda_1}\right)\bar{A}_2 + \frac{1}{(9+\bar{\mu}^2)^2}\right] \qquad (40)$$

$$\bar{a}_5 = -\frac{3\bar{\mu}^2}{16\lambda_1}(1+\text{Л})\left[\frac{1}{8} - \bar{A}_2\right] \qquad (41)$$

$$\overleftrightarrow{\psi}_3 = \sum_{k=1}^{Nk}3\bar{E}_k\bar{I}_k\bar{L}_kC_1^4 + \sum_{j=1}^{N_j}3\bar{E}_j\bar{I}_j\bar{L}_jC_2^4 + \sum_{k=1}^{Nk}3\bar{G}_k\bar{J}_k\bar{L}_kC_1^2S_1^2 + \sum_{j=1}^{N_j}3\bar{E}_j\bar{I}_j\bar{L}_j\,C_2^2S_2^2 \qquad (42)$$

ISSN 2229-5518

Where $\lambda = \frac{\text{ƀ}_3^*}{\beta}$ *and* $\lambda_1 = \frac{\text{ƀ}_4^*}{\text{ƀ}_3^*}$

Eliminating β and ƀ_2^* from Eqns. (3.64), (3.65), (3.66), the following equation was obtained.

$$\mathfrak{M}_1 \Phi^2 + \mathfrak{M}_2 \Phi + \mathfrak{M}_3 = 0 \quad (43)$$

Where

$$\mathfrak{M}_1 = \frac{1}{(1-Л^2)}\left[\frac{\omega_2\omega_3}{\eta_3^2}\left(\bar{a}_4 + \frac{\bar{a}_5}{\lambda} - \bar{A}_5\right)^2 + \frac{\eta_2}{\eta_3}\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda}\right)^2 - \left(\frac{\eta_2\omega_3}{\eta_3^2} + \frac{\omega_2}{\eta_3}\right) * \left(\bar{a}_4 + \frac{\bar{a}_5}{\lambda} - \bar{A}_5\right)\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda}\right)\right] \quad (44)$$

$$\mathfrak{M}_2 = -\frac{\bar{\mu}^2\bar{P}}{(1-Л^2)}\left\{\frac{2\omega_2\omega_3}{\eta_3^2}(\bar{a}_4 + \frac{\bar{a}_5}{\lambda})\left(\bar{a}_4 + \frac{\bar{a}_5}{\lambda} - \bar{A}_5\right) + \frac{2\eta_2}{\eta_3}\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda} - \bar{A}_5\right) - \left(\frac{\eta_2\omega_3}{\eta_3^2} + \frac{\omega_2}{\eta_3}\right)\left[\bar{A}_5^2 + 2\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda}\right)\left(\bar{a}_4 + \frac{\bar{a}_5}{\lambda}\right) - \bar{A}_5\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda} + \bar{a}_4 + \frac{\bar{a}_5}{\lambda}\right)\right]\right\} \quad (45)$$

$$\mathfrak{M}_3 = \frac{\bar{\mu}^4\bar{P}^2}{(1-Л^2)}\left[\frac{\omega_2\omega_3}{\eta_3^2}\left(\bar{a}_4 + \frac{\bar{a}_5}{\lambda}\right)^2 + \frac{\eta_2}{\eta_3}\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda} - \bar{A}_5\right)^2 - \left(\frac{\eta_2\omega_3}{\eta_3^2} + \frac{\omega_3}{\eta_3}\right)\left(\bar{a}_4 + \frac{\bar{a}_5}{\lambda}\right)\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda} - \bar{A}_5\right)\right] + \frac{{\eta_2}^2\omega_3^2}{\eta_3^2} - \frac{2\omega_2\omega_3\eta_2}{\eta_3} + \omega_2^2 \quad (46)$$

$$\omega_2 = \bar{A}_1\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda}\right) - \text{฿}_1\bar{A}_5 \quad (47)$$

$$\omega_3 = (\bar{A}_2 + \bar{A}_3\lambda + \bar{A}_4\lambda^2)\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda}\right) - \text{฿}_2\bar{A}_5\lambda^2 \quad (48)$$

$$\eta_2 = \bar{A}_1\left(\bar{a}_4 + \frac{\bar{a}_5}{\lambda}\right) - \bar{a}_1\bar{A}_5 \quad (49)$$

$$\eta_3 = (\bar{A}_2 + \bar{A}_3\lambda + \bar{A}_4\lambda^2)\left(\bar{a}_4 + \frac{\bar{a}_5}{\lambda}\right) - (\bar{a}_2 + \bar{a}_3\lambda^2)\bar{A}_5 \quad (50)$$

And

$$\Phi = \bar{\sigma}_b + {}^3/_2\,\bar{\sigma} \quad (51)$$

Equation (43) is the governing equation for determining the critical buckling stress of an internally pressurized thin cylindrical shell reinforced with angular stiffeners and loaded with eccentric compressive force or bending, where Φ is called minimum stress parameter for stiffened shell under bending and internal pressure.

4.0 RESULTS AND DISCUSSIONS

4.1 RESULTS

NUMERICAL EXAMPLES

The numerical analysis of this type of cylindrical shell was done by taking the following assumptions: $\bar{E}_k\bar{I}_k\bar{L}_k = \bar{E}_j\bar{I}_j\bar{L}_j$, $\bar{G}_k\bar{J}_k\bar{L}_k = \bar{G}_j\bar{J}_j\bar{L}_j$, $\Upsilon_1 = \Upsilon_2 = \Upsilon$ (*for* $\Upsilon = 10^0, 20^0, 30^0, 40^0, 45^0, 50^0, 60^0$), $\lambda = \lambda_1$, $m = 5$, $\bar{\mu} = 1$, $h = 0.05\, metre$, $\bar{P} = 2$ *and* $R = 2\, metres$. Using the governing equation in Eqn (43) and the notation described from Eqn (44) to Eqn (50), the following data shown in Table 1 and the corresponding graph in Fig. 4 were respectively obtained for different imperfect ratio, Л.

Table 1: Values of Buckling Stress Parameter, **Φ** for different imperfect ratio for internally pressurized thin cylinders reinforced with inclined stiffeners subjected to uniform bending

IMPERFECT RATIO, **Л**	BUCKLING STRESS PARAMETER, **Φ** OF STIFFENERS AT DIFFERENT ANGLES						
	10^0	20^0	30^0	40^0	45^0	50^0	60^0
0.1	7.6307	4.6271	1.4266	0.4038	0.3430	0.4785	1.2589
0.2	8.1084	5.3111	1.6910	0.4387	0.3665	0.5358	1.5150
0.3	8.4625	5.9047	1.9723	0.4819	0.3913	0.5865	1.7508
0.4	8.6674	6.3654	2.2466	0.5316	0.4167	0.6268	1.9430

0.5	8.6951	6.6554	2.4860	0.5863	0.4425	0.6552	2.0576
0.6	8.5097	6.7345	2.6572	0.6422	0.4684	0.6573	2.0488
0.7	8.0586	6.5516	2.7201	0.7031	0.4943	0.6360	1.8640
0.8	7.2535	6.0284	2.6262	0.7602	0.5202	0.5811	1.4580
0.9	5.9127	5.0181	2.3144	0.8118	0.5461	0.4846	0.8131

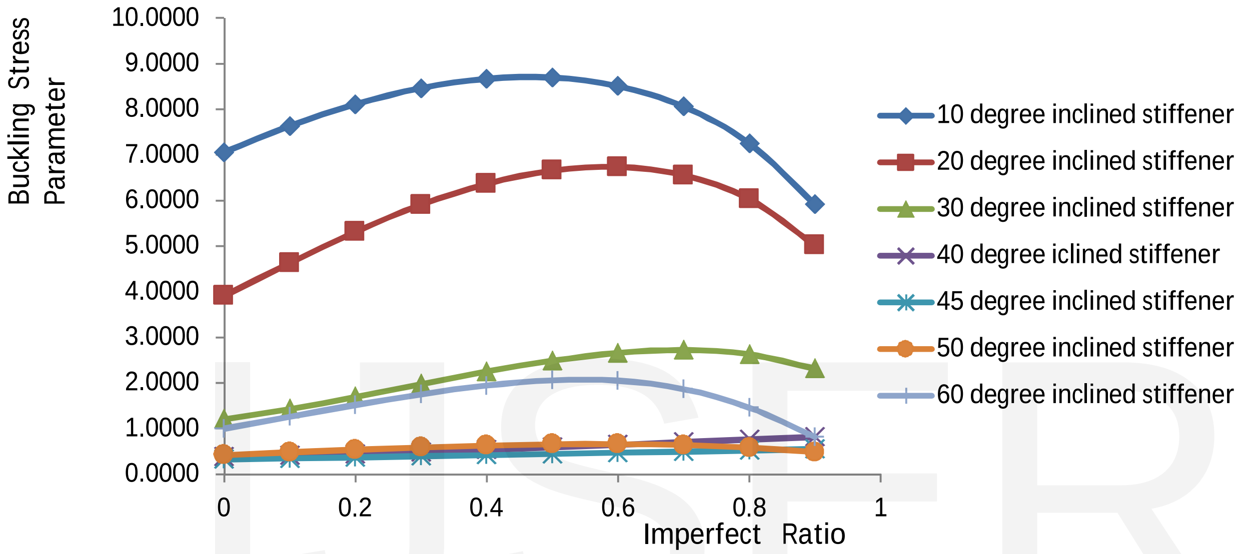


Fig. 4: Graph of Buckling Stress Parameter and Imperfect Ratio of Internally Pressurized Thin Cylinders under uniform bending

4.2 DISCUSSION OF RESULTS

The data in Fig. 4 showed that as imperfect ratio of stiffeners inclined at 10^0 and 60^0 respectively increases from 0.1 to 0.5, its buckling stress parameter increases. For stiffeners inclined at 20^0 and 50^0 respectively, there was progressive increase of their buckling stress parameter from imperfect ratio of 0.1 to imperfect ratio of 0.6. While, stiffeners inclined at 30^0 have progressive increase of their buckling stress parameter from imperfect ratio of 0.1 to imperfect ratio of 0.7.

However, stiffeners inclined at 40^0 and 45^0 respectively have progressive increase of their buckling stress parameter from imperfect ratio of 0.1 to imperfect ratio of 0.9. Buckling stress parameter is least at 45^0 inclined stiffeners and maximum at 10^0 inclined stiffeners for all imperfect ratios considered. The results in Table 1 and Fig.4 also showed that 10^0 inclined stiffeners is the most effective stiffener with maximum critical buckling stress at imperfect ratio of 0.5, while 45^0 inclined stiffeners is the least effective with the least critical buckling stress at imperfect ratio of 0.1

5.0 CONCLUSION

With reference to the results obtained in this research, engineers designing cylindrical shell structures with the aim of providing resistance to buckling would be able to select suitable inclined stiffeners for the structure under uniform bending.

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DEFINITION OF NOTATIONS

Notations	Meaning
$\in_x, \in_y$	Strains in x and y
w_0	Initial deflection
w	Total radial deflection
E	Young's modulus of elasticity of the shell
D	Flexural rigidity of the shell
h	Thickness of the shell
μ	Poisson ratio
Л	Imperfect ratio
R	Radius of the cylindrical shell
F	Airy's stress function
E_j	Young's modulus of elasticity of j^{th} stiffeners
E_k	Young's modulus of elasticity of k^{th} stiffeners
G_k	Shear modulus of k^{th} stiffeners
G_j	Shear modulus of j^{th} stiffeners
I_j	moment of inertia of j^{th} stiffeners
I_k	moment of inertia of k^{th} stiffeners
J_k	polar moment of inertia of k^{th} stiffeners
J_j	polar moment of inertia of j^{th} stiffeners
L	Length of the cylindrical shell
L_k	Length of k^{th} stiffeners
L_j	Length of j^{th} stiffeners
$\bar{L}_J$	Dimensionless length of j^{th} stiffeners
$\bar{L}_k$	Dimensionless length of k^{th} stiffeners
$\bar{E}_k$	Dimensionless Young's modulus of elasticity of k^{th} stiffeners
$\bar{E}_j$	Dimensionless Young's modulus of elasticity of j^{th} stiffeners
$\bar{G}_j$	Dimensionless shear modulus of j^{th} stiffeners
$\bar{G}_k$	Dimensionless shear modulus of k^{th} stiffeners
$\bar{I}_j$	Dimensionless moment of inertia of j^{th} stiffeners
$\bar{I}_k$	Dimensionless moment of inertia of k^{th} stiffeners
$\bar{J}_j$	Dimensionless polar moment of inertia of j^{th} stiffeners

$\bar{J}_k$	Dimensionless polar moment of inertia of k[th] stiffeners
m	Number of waves in axial direction
n	Number of waves in circumferential direction
γ_1	Inclination angle of the K[th] stiffeners parallel with y_1-line and normal to y_2^1-line
γ_2	Inclination angle of the j[th] stiffeners parallel with y_2- line and normal to y_1^1-line
$U_{T,j}$	Torsional strain energy for j[th] stiffeners inclined at angle, γ_2
$U_{T,k}$	Torsional strain energy for K[th] stiffeners inclined at angle, γ_1
$U_{b,j}$	Bending strain energy for j[th] stiffeners inclined at angle, γ_2
$U_{b,k}$	Bending strain energy for K[th] stiffeners inclined at angle, γ_1
U_b	Bending strain energy in the shell
U_e	Extensional strain energy in the shell
U_P	Strain energy due to internal pressure
U_m	Potential due to edge bending due to application of eccentric loading
σ_b	Bending stress
∇^4	Biharmonic operator
∇^2	Laplace operator
P	Internal pressure
$\bar{\mu}$	Wavelength ratio
$\bar{P}$	Dimensionless internal pressure
u, v, w	Components of displacements in x, y, z directions
x, y, z	Orthogonal coordinates on median surface of the shell
β	Dimensionless parameter that connect h, R and m
u, v, w	Components of displacements in x, y, z directions
x, y, z	Orthogonal coordinates on median surface of the shell
β	Dimensionless parameter that connect h, R and m
C_1	Cosine of angle γ_1
S_1	Sine of angle γ_1
C_2	cosine of angle γ_2
S_2	Sine of angle γ_2
λ , λ_1	Deflection parameters
N_K	Number of stiffeners in γ_1 -direction
N_J	Number of stiffeners in γ_2 –direction
$\prod$	Total strain energies
$\bar{\prod}$	Non-dimensional total strain energies
$\bar{U}$	Non-dimensional strain energy